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Firefly Synchronization in Ad Hoc Networks

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***Abstract*— In this article spontaneous synchronization observed in nature is applied to self-organized wireless networks. In South-East Asia huge swarms of fireflies emit light flashes in perfect synchrony. The underlying principle of this firefly synchronization scheme is reviewed and challenges related to the implementation in ad hoc networks are addressed. In particular, the effects of transmission delays and the constraint that a node cannot receive and transmit at the same time are studied.**

I. Introduction

As mobile communication becomes more and more ubiquitous and wireless links interconnect not only electronic devices but also everyday items, the need for self-organization in networking procotols becomes more and more apparent. With “self-organization” we mean that entities organize themselves in a distributed manner without the need for any external or central control entity. The entities interact directly with each other in a peer-to-peer fashion. In addition, each entity applies rather simple rules which then lead to sophisticated functionality of the overall system. This relation between microscopic and macroscopic behavior is also called “emergent behavior.”

Several interesting phenomena of self-organization and emergence can by observed in nature, one of the most prominent ones being the synchronization of blinking behavior of fireflies in South-East Asia [1]. While the origins of this phenomena are still not completely understood, a diligent mathematical model and theory has been presented in [2]. Clearly, an interesting research issue is to try to “learn from nature” and apply this theory to the time synchronization of entities in wireless networks.

Existing time synchronization protocols may impose prohibitive constraints when applied to a wireless environment. Synchronization of cellular systems may be achieved by the Global Positioning System (GPS), but in an indoor environment, it may not always be possible to receive the required signals. Furthermore, for sensor networks the implementation of a GPS receiver may be prohibitive, due to constraints in cost, power consumption and/or size. The Network Time Protocol (NTP) is used to maintain clock synchronized in the Internet and other distributed systems [3]. A hierarchy is defined, and messages emitted by master nodes are used by child nodes to synchronize. NTP was designed for wired networks, and is not well suited to rapid changes in the network topology [4]. With the Reference Broadcast Synchronization (RBS), receivers synchronize by exchanging the timestamp of the receive time of a reference broadcast signal [5]. Thus nodes need to exchange explicit messages, and the number of exchanged messages grows with the network size.

This short paper proposes a different approach based on the synchronization phenomena of fireflies, which does not require the exchange of an explicit message. We first review the original firefly synchronization algorithm. Next, we discuss problems in applying the algorithm directly to wireless networks. Finally, we outline a path to solve the problem and briely present our approach.

II. Firefly Synchronization

Fireflies can simply be abstracted as oscillators that emit a pulse of light periodically. This type of oscillators is referred to as “pulse coupled oscillators”, and are also used to study biological systems such as neurons and earthquakes [6]. This section describes how time synchronization is achieved in a decentralized fashion between these oscillators.

A. Mathematical Model

Pulse-coupled oscillators refer to systems that oscillate periodically in time and interact each time they complete an oscillation. This interaction takes the form of a pulse that is perceived by neighboring oscillators.

As a simple mathematical representation, a pulse-coupled oscillator is completely described by its phase function $\phi(t)$. This function evolves linearly over time until it reaches the threshold value ϕ_{th}. When this happens, the oscillator is said to fire, meaning that it will transmit a pulse and reset its phase. If not coupled to any other oscillator, it will naturally oscillate and fire with a period equal to T. Fig. 1(a) plots the evolution of the phase function during one period when the oscillator is isolated.

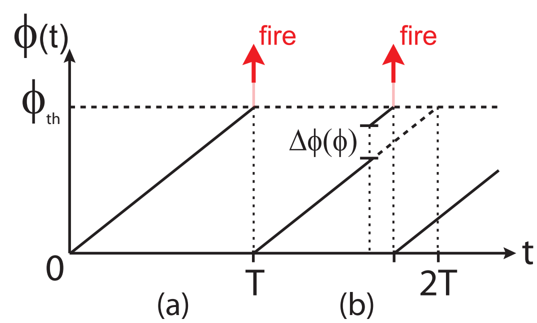

Fig. 1. Time evolution of the phase function

The phase function encodes the remaining time until the next firing, which corresponds to an emission of light for a firefly.

B. Synchronization of Pulse-Coupled Oscillators

Mirollo and Strogatz analyzed spontaneous synchronization phenomena and also derived a theoretical framework based on pulse-coupled oscillators for the convergence of synchrony [2].

When coupled to others, an oscillator is receptive to the pulses of its neighbors. When receiving such a pulse, it will instantly increment its phase by an amount that depends on the current value:

$$\phi \rightarrow \phi + \Delta\phi \text{ when receiving a pulse}$$

Fig. 1(b) plots the time evolution of the phase when receiving a pulse. The received pulse causes the oscillator to fire early.

The phase increment $\Delta\phi$ depends on the current phase, and it is determined by the Phase Response Curve (PRC), which was chosen to be linear in [2]:

$$\phi + \Delta\phi = \min\left(\alpha \cdot \phi + \beta, 1\right) \quad \text{with} \quad \begin{cases} \alpha = \exp(b \cdot \epsilon) \\ \beta = \frac{\exp(b \cdot \epsilon) - 1}{\exp(b) - 1} \end{cases} \tag{1}$$

where b is the dissipation factor and ϵ is the amplitude increment. Both factors determine the coupling between oscillators. The threshold ϕ_{th} is normalized to one.

It was shown in [2] that if the network is fully meshed and $b > 0$ and $\epsilon > 0$, the system always converges, i.e. all oscillators will fire as one independently of initial conditions. The time to synchrony is inversely proportional to the product $b \cdot \epsilon$.

Interestingly the synchronization scheme relies on the instant of arrival of a pulse, and receivers adjusting their phases when detecting this pulse. Interference in the typical way is not observed, and two pulses emitted simultaneously can superimpose constructively. This helps a faraway receiver to detect the superimposed pulses, and to synchronize with the rest of the network. Furthermore, it has been shown that for wireless networks, spatial averaging can be beneficially used to bound the synchronization accuracy to a constant, making the protocol scalable [7].

III. Application to Wireless Systems

The synchronization scheme described previously can be directly applied to wireless systems with some adjustment needed to combat the propagation delay. When generalizing this model to wireless environments, different delays need to be taken into account and affect the achievable accuracy.

A. Synchronization through Pulses

Hong and Scaglione [8] adapted the theoretical framework of [2] to wireless sensor networks. In their work the synchronization protocol was modified to better model a wireless communication system by taking into account propagation delays [8].

If a propagation delay T_0 occurs between two pulse coupled oscillators, the system can become unstable [9]. The pulse of one oscillator could cause the other oscillator to transmit after T_0, and this transmitted pulse causes the first oscillator to fire again after T_0, and so on.

To avoid this avalanche effect a refractory period of duration T_{refr} needs to be added after transmission. During this period, the phase function of a node stays equal to 0 and is not modified if receiving a pulse [8]. Stability is maintained if echos are not received, which translates to a condition on T_{refr}:

$$T_{\text{refr}} > 2 \cdot T_0 \tag{2}$$

With the introduction of the refractory state, the accuracy of the synchronization scheme is equal or smaller to twice the maximum refractory delay.

In [8], once the sensor network is synchronized, the protocol can also be used to propagate information in a similar fashion to the pulse-position modulation. If a node wants to transmit data, it will spontaneously shift the established reference instant, thus breaking the equilibrium and also forcing surrounding nodes to shift their reference instant. This global shift can be used to convey information to an observer that will detect the offset in the reference instant.

This scheme implies that a receiver is able to immediately detect a single pulse of infinitely small width, and no decoding is done by the receiver. The following section will present the different delays that prevent direct application of the firefly synchronization scheme when considering communication with consequent transmission delays.

B. Delays in Wireless Systems

In the synchronization model of pulse-coupled oscillators described previously, it is assumed that communication is done through pulses and that a pulse is instantly received and decoded by other oscillators. In a wireless environment solitary pulses are hardly used alone as they are virtually impossible to detect. More realistically a sequence of pulses or a burst of duration T_{Tx} is to be considered for the synchronization scheme. During this time a node is not able to receive.

Once transmitted, this message will not be instantly received as some propagation delay T_0 occurs. After the message has propagated and been received, some processing time is required to correctly declare that a synchronization message has been received. This results in a decoding delay T_{dec}. The time taken by a node to completely receive and decode a synchronization message is changed. It now requires $(T_{\text{Tx}} + T_{\text{dec}})$.

Alltogether, four delays need to be taken into account to model the synchronization strategy to a wireless network:

- T_0: Propagation delay - time taken for a burst to propagate from the emitting to the receiving node. This time is proportional to the distance between two nodes.
- T_{Tx}: Transmitting delay - length of the burst. A node cannot receive during this time.
- T_{dec}: Decoding delay - time taken by the receiver to decode a burst.
- T_{refr}: Refractory delay - time necessary after transmitting to maintain stability.

These delays are the most significant difference from the Mirollo and Strogatz model, which assumes no propagation delay, an infinitely short transmission time and no decoding delay [2]. The total delay is defined by:

$$T_{\text{del}} = T_0 + T_{\text{Tx}} + T_{\text{dec}} \tag{3}$$

This total delay represents the inherent time difference between the beginning of the transmission of a synchronization burst and its successful reception. To make matters worse, during transmission it is not possible to receive. As a consequence a "blind spot" of duration T_{del} appears in which nodes cannot listen to the network. Within a blind spot no mutual coupling between nodes can occur, which implies that the attainable synchronization accuracy is lower bounded by T_{del}. If considering a transmission technology with a short transmitting pulse, such as UWB systems, the attained accuracy might be sufficient, as T_{Tx} would be negligible [8]. Unfortunately, for many transmission techniques, where the time for one symbol block, T_{Tx}, cannot be assimilated as a pulse, such an accuracy is clearly unacceptable. Therefore, there is a need to modify the synchronization strategy.

IV. Time Advance Strategy

One strategy to combat the loss of accuracy is for the transmitter to delay its transmission for a certain time equal to:

$$T_{\text{wait}} = T - (T_{\text{Tx}} + T_{\text{dec}}) \tag{4}$$

where T denotes the synchronization period. With this approach, if T_0 is neglected, the receiver will increment its phase exactly T seconds after the transmitter has fired.

This scheme modifies the natural oscillatory period of an oscillator, which is now equal to $2 \cdot T$. The time during which the phase function will increment is reduced by the waiting, transmitting and refractory delays. It is now equal to:

$$T_{\text{Rx}} = 2 \cdot T - (T_{\text{wait}} + T_{\text{Tx}} + T_{\text{refr}}) \tag{5}$$

Fig. 2 sums up this strategy for two oscillators that are already synchronized.

At instant 0, oscillator 1 reaches ϕ_{th}. It waits until $t_1 = T_{\text{wait}}$ before starting to transmit a synchronization burst. At $t_2 = T_{\text{wait}} + T_{\text{Tx}} + T_{\text{dec}} = T$, oscillator 2 has successfully received and decoded the burst. As the two oscillators are already synchronized, it will follow the same scheme as oscillator 1, and wait until $t_3 = T + T_{\text{wait}}$ before transmitting.

More generally, a transmitter waits for a time equal to $T_{\text{wait}} = L \cdot T - (T_{\text{Tx}} + T_{\text{dec}})$, with $L \geq 1$ being a positive integer. Instead of directly firing, i.e. transmitting a synchronization burst, the transmitter waits for L periods; taking into account the time span which is consumed by transmitting. This corresponds to a timing advance strategy, which ensures that other nodes do not observe the unavoidable transmission and decoding delay.

For a system of N oscillators firing instants are initially randomly distributed over a period of $2 \cdot T$. Each oscillator will follow the same rules of waiting before transmitting and

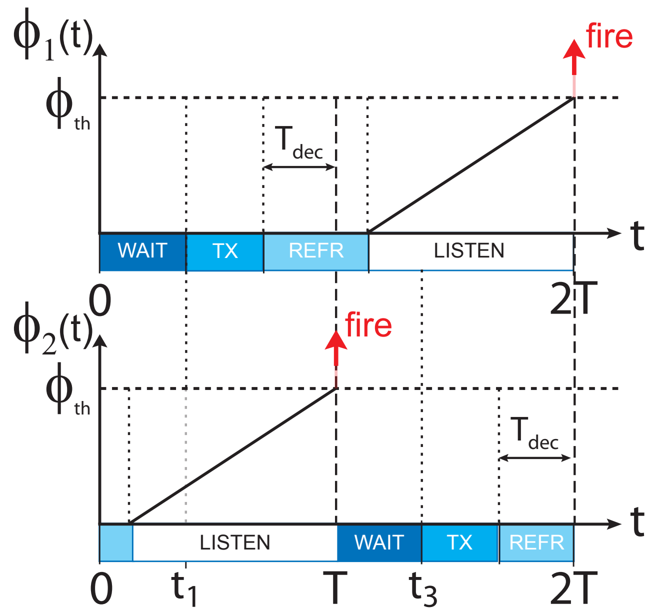

Fig. 2. Description of the timing advance transmit strategy for two periods

incrementing its phase when fully receiving and decoding a message while listening. Over time the oscillators will split into two groups, each group firing T seconds apart and helping each other to synchronize. Therefore T is still used as the reference synchronization period.

Thanks to the new transmitting strategy the accuracy of synchronization is longer limited by T_{del}. Successful synchronization is therefore declared when firing instants are spread over a time interval that is equal or smaller than T_0.

A. Results

Figs. 3 and 4 respectively plot the synchronization rate and the mean time to synchrony, depending on the coupling factors b and ϵ defined in (1), in a fully meshed network with 30 nodes. Results are shown for timing advance synchronization scheme with $L = 1$. The simulation is done by decomposing each period T into 1000 steps, and calculating the corresponding state and interactions for each node at each step. The initial conditions correspond to the case where all nodes have randomly distributed state variables. The synchrony rate on Fig. 3 was obtained by running the time advance strategy on 1000 sets of initial conditions, and successful synchronization is detected when the timing accuracy does not exceed the propagation delay T_0.

It is seen that it is possible to identify combinations of b and ϵ for which global synchronization is always achieved, and the average time for the network to converge can be lower than 15 periods.

V. Conclusion

With the proposed synchronization scheme, a level of accuracy can be achieved that is only limited by the propagation delay. Communication through pulses is no longer required,

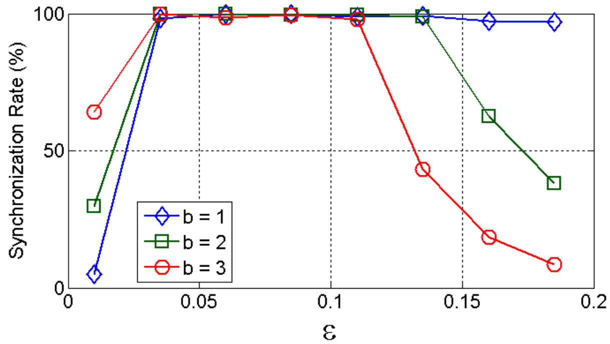

Fig. 3. Synchronization rate for various coupling factors b and ϵ in a fully meshed network with 30 nodes. The system parameters are set to $T_{\mathrm{Tx}} = 0.15 \cdot T$ and $T_{\mathrm{dec}} = 0.35 \cdot T$

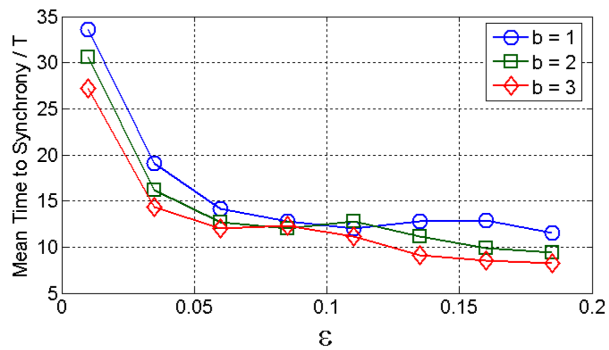

Fig. 4. Mean Time to Synchrony, $\bar{T}$, for various coupling factors b and ϵ in a fully meshed network with 30 nodes. The system parameters are set to $T_{\mathrm{Tx}} = 0.15 \cdot T$ and $T_{\mathrm{dec}} = 0.35 \cdot T$

and synchrony of a system of oscillators can still be ensured. The simplicity and generality of the synchronization scheme makes its implementation very appealing. If all nodes cooperate, synchrony can be reached within 15 periods. Once nodes have agreed on a common time scale, they are then able to use the full time slot to communicate in a synchronous manner.

While the timing advance strategy is very simple, the waiting time imposes additional delays, raising the constraints to achieving convergence and stability. In a not fully-meshed multi-hop network, however, the situation is more complicated. Due to the fact that the timing advance strategy effectively establishes $(L + 1)$ groups, formations might occur where one node is surrounded by nodes which are all in the same group. This may result in a "deafness effect", where a local group of nodes all transmit at similar time instants, which implies that these nodes cannot hear each other. While for a fully meshed network the probability that all local nodes are within one group tends to zero, the deafness effect causes severe problems for meshed networks, and is a suitable topic for further research.

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